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Spectral information support for unmanned ground vehicles

Vasily P. Larshin¹⁾ORCID: <https://orcid.org/0000-0001-7536-3859>; vasilylarshin@gmail.com. Scopus Author ID: 7801669969Denys O. Shapovalov¹⁾ORCID: <https://orcid.org/0009-0007-4125-8875>; 388839@as.op.edu.uaIhor V. Kishianus²⁾<https://orcid.org/0000-0001-7838-5607>; myhouse72@ukr.netI Yu. Zlepko²⁾ORCID: <https://orcid.org/0009-0002-2582-0083>; zlepko@meta.uaOleksandr V. Maltsev²⁾<https://orcid.org/0000-0001-7377-5972>; mav1946fish@gmail.com¹⁾ Odesa Polytechnic National University, 1, Shevchenko Av. Odesa, 65044, Ukraine²⁾ Military Academy (Odesa-city), Fontans'ka doroha, 10, 65009, Ukraine

ABSTRACT

The paper investigates the application of Parseval's theorem to digital signal processing and its use for relating the mean-square value of a signal to the integral of its power spectral density. Particular attention is given to the relationship between the square of the root-mean-square amplitude and the area under the one-sided power spectral density curve, which represents the total mean-square power of the signal over the analyzed frequency range. A controlled multi-harmonic signal is used to examine how sampling conditions, additive noise, and spectral estimation affect the preservation and interpretation of signal power. Several computational cases are considered to analyze the influence of sampling at the Nyquist boundary, increased sampling frequency, stochastic noise, and spectral smoothing by Welch's method. The results demonstrate that Parseval's theorem provides an exact power-consistency relation for the discrete signal represented by the sampled data, but this consistency alone does not guarantee faithful representation of the original continuous physical signal. Adequate sampling frequency, measurement bandwidth, and spectral estimation methodology are therefore essential for physically meaningful spectral analysis. The study also demonstrates that the integrated power spectral density can be used as a quantitative criterion for verifying the consistency of time-domain and frequency-domain calculations. The relationship between Parseval's theorem and the Wiener-Khinchin theorem is clarified by distinguishing the conservation of mean-square quantities from the statistical description of stationary random processes. The developed signal-processing approach is further considered as an information-support framework for vibration monitoring of unmanned ground vehicles, where measured vibration signals can be transformed into frequency-domain representations for subsequent monitoring, diagnostics, and condition assessment. Parseval's theorem is proposed as an internal consistency check for this transition from measured time-domain vibration data to their spectral representation.

Keywords: Digital Signal Processing (DSP); Parseval's Theorem; Power Spectral Density (PSD); Root-Mean-Square (RMS); Nyquist Boundary; Phase Dropout; Welch's Method; Area Under the Curve (AUC); Unmanned Ground Vehicle (UGV)

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1. Introduction

For unmanned ground vehicles, spectral information can be considered not only as a means of vibration monitoring and technical diagnostics, but also as a broader component of information support for vehicle operation and control. Physical signals generated by the vehicle, its propulsion and transmission systems, interaction with the terrain, and motion of the vehicle body may contain information about both the current operating condition and the external environment. Their spectral representations can reveal periodic components, characteristic frequency bands, resonant phenomena, changes in dynamic behavior, and other features that may be difficult to identify from time-domain data alone. Consequently, spectral analysis can provide a common methodological framework for processing different types of physical signals relevant to unmanned ground vehicle operation.

This approach also creates a basis for extending spectral information support from monitoring and diagnostics toward adaptive control. In a closed-loop control system with negative feedback,

measured physical variables and their deviations from reference values can be transformed into spectral characteristics and used as additional information for assessing the dynamic state of the controlled object. For example, spectral components of acceleration, angular motion, suspension response, or tracking error may indicate the presence of periodic disturbances, resonant modes, or undesirable oscillations in the control process. In combination with conventional feedback signals, such information can potentially be used for adaptive modification of control parameters, vehicle speed, motion mode, or other control actions. Thus, the spectral domain may serve as an additional information layer between physical measurements and decision-making algorithms rather than being limited to post-processing and diagnostic purposes.

An important feature of this concept is that the analyzed signal does not have to originate exclusively from the propulsion system or from mechanical vibration. The interaction between an unmanned ground vehicle and the terrain also generates spatially distributed disturbances. In particular, the surface profile along the vehicle trajectory can be represented as a function of spatial coordinate, and its spectral characteristics can describe the distribution of surface irregularities over different spatial scales. When vehicle velocity is known, spatial-frequency characteristics can additionally be related to temporal excitation frequencies acting on the vehicle. This provides a natural connection between surface characteristics, vehicle dynamics, and the information used by control algorithms.

A related problem is encountered in surface metrology, where the measured distribution of micro-irregularities along a trace is analyzed to separate the actual surface texture from the influence of geometric form, installation errors, and other low-frequency components. After appropriate preprocessing and removal of the constant component, the resulting signal can be analyzed in the time or spatial domain and subsequently transformed into the frequency domain. Such an approach illustrates the general principle underlying the present study: a physical signal is first represented in a suitable discrete form, then transformed into the spectral domain, where its components can be quantitatively analyzed and interpreted. In this context, Parseval's theorem provides an important mathematical link between the mean-square characteristics of the signal in the original domain and its spectral representation.

The same principle can subsequently be extended to other signals associated with unmanned ground vehicle operation, including vehicle-body accelerations, suspension responses, drivetrain oscillations, road-surface irregularities, and control-system errors. Therefore, the spectral approach considered in this study is intended as a general information-processing framework whose application can be expanded from signal analysis and condition monitoring toward the development of intelligent monitoring and adaptive control systems for unmanned ground vehicles.

In the fields of mechanical engineering, surface metrology, and structural health monitoring, extracting reliable statistical parameters from continuous physical processes requires a seamless analytical link between the time and frequency domains. The foundational framework for modeling stochastic engineering profiles was established in the classic work by P. Ranganatha Nayak [1]. Nayak pioneered the application of two-dimensional Gaussian random process theory to characterize solid surface roughness. By utilizing spectral analysis, he proved that critical micro-roughness parameters – such as the spatial density of summits, mean profile slope, and peak curvature – are entirely governed by the spectral moments (m_0, m_2, m_4) derived from the “area under the curve” (AUC) of the spatial Power Spectral Density (PSD) function.

In modern digital data acquisition systems, this spatial characterization approach is completely analogous to evaluating temporal physical processes, such as machinery vibration signatures over time. In temporal signal processing, the zero-order spectral moment (m_0) represents the total variance or the square of the root-mean-square amplitude (RMS) R_q^2 of the signal. The mathematical engine that guarantees the equivalence of this energy metric across domains is Parseval's theorem (closely related to the Wiener-Khinchin theorem). Parseval's theorem dictates that the total mean power of a process remains invariant whether it is integrated as a mean-square value over an infinite time horizon or as an area under the continuous frequency spectrum [2].

A cornerstone of statistical signal processing and harmonic analysis is the ability to map physical characteristics of a process seamlessly between the temporal and spectral domains. According to Parseval's theorem, the total average power of a process remains constant whether it is calculated as the mean square value over an infinite time interval or as the area under the curve of a continuous frequency spectrum. This formulation fundamentally treats power conservation as a unitary property of the Fourier transform. However, when analyzing complex, real-world signals, Parseval's theorem shares a deep conceptual intersection – yet maintains a distinct boundary – with the Wiener–Khinchin theorem. Understanding the subtle similarities and operational differences between these two frameworks is crucial for robust spectral modeling.

The primary similarity between Parseval's and the Wiener–Khinchin theorems lies in their shared objective: both serve as mathematical bridges establishing that energy or power metrics are invariant to domain transformations. Both theorems dictate that the total energy (or power) observed in the time domain is perfectly preserved when mapped onto the continuous frequency axis. Furthermore, both methods rely heavily on the operational mechanics of the Fourier transform to extract spectral distributions.

Despite this common ground, the two theorems diverge significantly in their mathematical prerequisites and the types of signals they accommodate. Parseval's theorem operates directly on deterministic, square-integrable functions (L^2 space) possessing finite energy. Conversely, stochastic fluctuations, such as environmental noise or continuous random telemetry, possess infinite total energy over an infinite time horizon, causing their direct Fourier transforms to diverge. To resolve this, the Wiener–Khinchin theorem shifts the focus from the signal itself to its statistical properties, specifically establishing that the power spectral density (PSD) of a wide-sense stationary random process is the Fourier transform of its autocorrelation function. Consequently, while Parseval's identity directly integrates the squared magnitude of the deterministic signal's spectrum, the Wiener–Khinchin theorem maps the statistical correlation of a random process across time shifts directly into the frequency domain. This distinction allows researchers to generalize the preservation of total average power from strictly bounded, deterministic signals to continuous, stochastic phenomena [3].

However, when moving from continuous theoretical equations to discrete digital signal processing (DSP), engineers encounter practical hardware constraints, such as finite sampling rates, digital quantization, and stochastic background noise. While Parseval's algebraic equality remains strictly true for any isolated discrete digital array, its capacity to faithfully preserve the true energy budget of the underlying continuous physical process depends heavily on these parameters.

This paper systematically investigates the physical essence, mathematical derivation, and practical limitations of Parseval's theorem. Using a controlled multi-harmonic synthetic vibration signal, we evaluate how sampling frequency selections, zero-crossing synchronization, Gaussian noise injection, and advanced spectral smoothing via Welch's method alter both the visual topology of the spectrum and the convergence precision of the energy conservation law.

2. Literature review

In theoretical mechanics and descriptive geometry, one operates with the concepts of the “ideal plane”, the “ideal cylinder”, or the “sphere”. However, in applied physics and mechanical engineering, such abstractions are inapplicable. Any real surface of a solid body, obtained as a result of technological processing or operational service, represents a complex spatiotemporal assembly of deviations from the nominal (drawing-specified) geometric form.

The totality of these deviations, considered within a certain scale range, has received the designation surface texture in international standardization (ISO 4287, ANSI/ASME B46.1). Texture constitutes an informative parameter, since it accumulates the kinematics and dynamics of the cutting process (tool marks, vibrations), as well as the physical-mechanical properties of the material (susceptibility to plastic deformation, strain hardening) and the conditions of friction and lubrication in assemblies. Of fundamental importance is the fact that texture possesses a hierarchical

and even multiscale structure [4]. This signifies that on one and the same surface region, geometric imperfections of different orders exist simultaneously.

According to the standard, the complete surface profile (P) is mathematically decomposed into three independent components (1):

$$P(x) = F(x) + W(x) + R(x), \quad (1)$$

where $F(x)$ represents form error (the largest deviations, caused by machine tool deformations or thermal expansion), $W(x)$ represents waviness (intermediate-scale periodic deviations), and $R(x)$ represents roughness (micro-irregularities superimposed upon the waviness) [5].

It is precisely the separation of these components that constitutes the key procedure prior to the calculation of numerical parameters. The separation is accomplished by means of filters (for example, the Gaussian filter or spline filters), which act as frequency selectors: they suppress the long-wave components of form and isolate the short-wave components of roughness.

Waviness is defined as a set of periodic irregularities, the distance between adjacent peaks of which (the spacing) substantially exceeds the distance between roughness peaks. The ratio of waviness spacing to roughness spacing is typically no less than 3:1, and the height parameters of waviness (W_a, W_z) are an order of magnitude larger than the corresponding roughness parameters [4].

Surface roughness is defined as the finest irregularities of a real surface, considered within the assessment length (l_r), which form a relief conditioned by the material properties and the geometry of the cutting edge of the tool.

In engineering practice, it is conventional to associate the range of roughness values with the method of machining. For example, R_a 0.01 – 0.1 μm may be in superfinish grinding, polishing, honing, while $R_a > 12.5$ μm takes place in casting and stamping. The surface structure is comparable with geological formations (by analogy with pebbles, stones, and boulders, but at the microscale) [6]. Under Vibrational Wear the surface profile transforms from “regular” (tool marks) to “irregular” (wear) with increased bearing area and asymmetry of peaks/valleys [7]. Thus, the wear profile constitutes an integral response of the material to vibrational loading, which cannot be described by height parameters alone but requires a more refined mathematical apparatus.

Height parameters constitute the most widespread and intuitively understandable category. They answer the question: “How large are the irregularities in the vertical direction?”

For a long time, the parameter R_a (arithmetic mean deviation of the profile) dominated in industry. Parameter R_a is defined as the arithmetic mean of the absolute values of the deviations of the profile ordinates from the mean line (that is without a DC component) within the assessment length. For a continuous profile $y(x)$ on assessment length L :

$$R_a = \frac{1}{L} \int_0^L |y(x)| dx. \quad (2)$$

For a discrete digital profile consisting of N points with ordinates y_i :

$$R_a = \frac{1}{N} \sum_{i=1}^N |y_i|. \quad (3)$$

Geometrically, if we shade all areas between the profile and the mean line (both above and below), the total area of all shaded regions divided by the profile length L gives R_a :

$$R_a = \frac{\text{Total area of shaded regions}}{L}. \quad (4)$$

Parameter R_a represents the mean absolute height of the micro-irregularities. If $R_a = 0.8$ μm , this means that on average, each point of the profile deviates from the mean line by 0.8 micrometers.

However, it possesses a fundamental deficiency: it is insensitive to the shape of the height distribution. Two profiles with completely different numbers of sharp peaks and deep valleys may yield the same R_a value. For tribology, sealing, and strength, this is insufficient.

The root-mean-square roughness (RMS Roughness) is a more fundamental statistical parameter than R_a . It is the root-mean-square deviation (standard deviation) of the profile ordinates from the mean line (that is without a DC component) within the assessment length [5].

For a continuous profile (5):

$$R_q = \sqrt{\frac{1}{L} \int_0^L y^2(x) dx}. \quad (5)$$

For a discrete profile (6):

$$R_q = \sqrt{\frac{1}{N} \sum_{i=1}^N y_i^2}. \quad (6)$$

The squaring operation provides three fundamental advantages: 1) differentiability, i.e., the function y^2 is smooth and differentiable everywhere, unlike $|y|$, which has a derivative discontinuity at zero. This is important for analytical optimization methods; 2) energy analogy, i.e., in physics, the square of a quantity is often proportional to energy (R_q^2 is proportional to the mean energy of the micro-irregularities and used to compute the energy of an oscillatory process or the power of a signal); 3) statistical consistency i.e., in probability theory, the root-mean-square deviation is the most informative measure of the scatter of a random variable [5]. Besides, in contrast to R_a , where we sum the absolute values (moduli) of deviations, in R_q we square the deviations.

This endows the R_q parameter with the following critical properties:

1) quadratic weighting, i.e., large deviations (sharp peaks and deep valleys) contribute disproportionately more to the R_q value than numerous small irregularities;

2) correlation with friction, i.e., in static contact, the real area of contact is proportional to the mean square of the height of the asperities [8].

Some distinctions between R_q and R_a are shown in Table 1.

Table 1. Key Distinctions Between R_q and R_a

Characteristic	R_a	R_q
Operation	Modulus (y)	Square (y^2)
Weight of large deviations	Proportional	Quadratic
Sensitivity to outliers	Low	High
Statistical meaning	Mean absolute deviation	Standard deviation

Source: compiled by the authors

Numerical Example [7]:

Let the profile consist of 5 points: $y = [1, -2, 3, -1, 2]$. The mean line is drawn such that $\sum y_i = 0$. We have: $1 - 2 + 3 - 1 + 2 = 3 \neq 0$. We now center the function y (which has a DC component) by introducing a new function $y' = y - \bar{y}$ (without a DC component), where $\bar{y} = 3/5 = 0.6$. Then we obtain: $y' = [0.4, -2.6, 2.4, -1.6, 1.4]$.

Calculation of R_a gives:

$$R_a = \frac{|0.4| + |-2.6| + |2.4| + |-1.6| + |1.4|}{5} = \frac{0.4 + 2.6 + 2.4 + 1.6 + 1.4}{5} = \frac{8.4}{5} = 1.68 \mu\text{m}. \quad (7)$$

Calculation of R_q :

$$\begin{aligned}
 R_q &= \sqrt{\frac{0.4^2 + (-2.6)^2 + 2.4^2 + (-1.6)^2 + 1.4^2}{5}} = \sqrt{\frac{0.16 + 6.76 + 5.76 + 2.56 + 1.96}{5}} = \\
 &= \sqrt{\frac{17.2}{5}} = \sqrt{3.44} \approx 1.855 \mu\text{m}.
 \end{aligned}
 \tag{8}$$

Thus, $R_q \approx 1.855$ is larger than $R_a \approx 1.68$, because the larger deviations (particularly -2.6 and 2.4) received greater weight upon squaring.

While scalar parameters provide a single number for the characterization of the surface, statistical functions of surface topography provide a complete picture of the height distribution and correlation. We now consider the power spectral density (PSD) [9]. PSD is arguably the most informative statistical function for surface analysis. It answers the question: "What contribution to the total roughness is made by irregularities with different spatial frequencies (wavelengths)?"

By analogy with acoustics: PSD is the "frequency spectrum" of the surface. Just as an acoustic signal can be decomposed into notes of different pitches, the surface profile can be decomposed into sinusoidal waves of different wavelengths.

For a continuous profile we have:

$$PSD(F) = \lim_{L \rightarrow \infty} \frac{1}{L} \left| \int_0^L y(x) e^{-i2\pi Fx} dx \right|^2. \tag{9}$$

where: F is the spatial frequency (μm^{-1}); L is the profile length; the integral is the Fourier transform of the profile.

Let there be N points with spacing Δx . The total length $L = N \cdot \Delta x$. Then for a discrete digital profile:

$$PSD\left(\frac{k}{L}\right) = \frac{1}{N\Delta x} \left| \sum_{j=1}^N y_j e^{-i2\pi k j / N} \right|^2. \tag{10}$$

where $k = 1, 2, \dots, N/2$ are integers corresponding to frequencies; the spatial frequency $F = k/L$; the wavelength $\lambda = 1/F = L/k$.

It should be noted that the computation of PSD requires the use of the Fast Fourier Transform (FFT) algorithm to reduce the computational time from $O(N^2)$ to $O(N \log N)$ [9]. The notation $O(N^2)$ denotes quadratic complexity, while $O(N \log N)$ denotes quasi-linear complexity. The latter is achieved by the FFT algorithm, which substantially accelerates spectral computations compared to the direct Discrete Fourier Transform (DFT).

For the interpretation of PSD let's show the illustrative examples. For the first example, let it be a sinusoidal surface. This is a reference surface with a regular structure (for example, a diffraction grating). The PSD of such a surface exhibits a pronounced peak at the fundamental frequency $F = 0.01 \mu\text{m}^{-1}$ with the wavelength: $\lambda = 1/F = 100 \mu\text{m}$. The PSD reveals the dominant frequency corresponding to the grating period. However, higher harmonics (at frequencies that are multiples of the fundamental) are also visible, which indicate imperfections in the sinusoidal form (for example, flat summits or sharp valleys) [10]. For the second example, let it be a ground surface. For a real ground surface, the PSD exhibits a monotonic character – it increases smoothly with increasing wavelength (decreasing frequency). This signifies that the large-scale irregularities possess greater amplitude than the fine-scale ones [11].

Having established the fundamental behavior of PSD for both sinusoidal and ground surfaces, it is instructive to examine the broader theoretical framework that governs the spectral characteristics of surfaces across all scales and processing methods.

In the course of a systematic analysis of power spectral densities (PSDs) for an extensive multitude of surfaces, encompassing the range from high-precision polished steels to extreme

topographical objects, including the lunar surface, Sayles and Thomas established a fundamental empirical generalization that has attained the status of a universal law. According to this law, the PSD increases monotonically with increasing spatial wavelength for all types of surfaces irrespective of the specific method of their finishing treatment. This discovery has fundamental significance for surface metrology, since it indicates that the hierarchical structure of irregularities, extending from micron-scale grinding marks to macroscopic deviations of form and waviness, constitutes an immanent property of any real surface obtained under conditions of technological production. The physical nature of this phenomenon is determined by the fact that any process of mechanical machining is accompanied by random fluctuations of external parameters, among which are vibrations of the elements of the technological system, non-uniformity of abrasive grains with respect to size and shape, as well as the structural non-uniformity of the material being machined. The combined action of the indicated stochastic factors forms a continuous spectrum of irregularities, in which the amplitude of the harmonic components naturally increases with increasing spatial scale [11].

While the universal law provides a robust theoretical description of surface spectral behavior, its empirical verification and practical application are critically dependent on the measurement capabilities of the instrumentation employed.

On the presented illustrative material, in particular on the graph of the PSD for a ground surface, a sharp drop in PSD values is clearly observed in the region of low frequencies approaching the zero value of the spatial frequency. It is important to emphasize that this behavior does not reflect the physical properties of the surface itself, but rather represents an artifact caused by the instrumental limitations of the measuring system. The reason for this is that the effective bandwidth of the instrument is limited by the maximum length of the trace available during scanning. In accordance with the fundamental principles of measurement theory, the instrument is fundamentally incapable of registering spectral components whose spatial wavelength exceeds the trace length, since the correct reproduction of such a harmonic requires that the assessment length encompass at least one complete period of the profile oscillation. The consequence of this is the artificial truncation of the PSD in the low-frequency region, that is, at large wavelengths, which may lead to erroneous interpretation of the absence of long-wave components in the surface topography. Hence follows the critically important practical requirement for conducting spectral analysis of surfaces: in the interpretation of PSD results, it is mandatory to take into account both the trace length and the frequency range of the measuring instrument; otherwise, there arises the danger of systematic underestimation of the contribution of large-scale irregularities to the overall surface profile [12].

In the preceding sections, the power PSD was considered as a tool for analyzing the frequency composition of surface profiles. However, for a complete understanding of the relationship between the spectral and amplitude descriptions of surfaces, it is necessary to consider a fundamental theorem that establishes a quantitative connection between these two approaches – Parseval's theorem. This theorem, named after the French mathematician Marc-Antoine Parseval (1755–1836), who first formulated it in 1799 in the context of Fourier series, is one of the cornerstones of modern signal processing and spectral analysis theory [13]. The generalization of this theorem to the case of continuous signals and integral Fourier transforms was performed considerably later, in particular in the works of Rayleigh (1889) and Plancherel (1910), who proved it for square-integrable functions [14].

Parseval's theorem in its classical form states that the total energy of a signal in the time (or spatial) domain equals the total energy of its spectrum in the frequency domain, up to a constant factor [14]. For a continuous signal $y(x)$, defined on the entire real axis, belonging to the space $L^2(\mathbb{R})$, that is, satisfying the condition $\int_{-\infty}^{\infty} |y(x)|^2 dx < \infty$, and its Fourier transform $Y(f) = \int_{-\infty}^{\infty} y(x) e^{-i2\pi f x} dx$, Parseval's theorem takes the form:

$$\int_{-\infty}^{\infty} |y(x)|^2 dx = \int_{-\infty}^{\infty} |Y(f)|^2 df. \quad (11)$$

This statement is a consequence of the more general Plancherel theorem, which states that the Fourier transform is a unitary operator in $L^2(\mathbb{R})$ up to normalisation [14]. Unitarity implies the preservation of the inner product and, consequently, the norm (energy) of the signal [14]. In practical metrology, where the profile is always represented as a discrete data array y_i (where $i = 0, 1, \dots, N-1$), the discrete Fourier transform (DFT) is used: $Y_k = \sum_{i=0}^{N-1} y_i e^{-i2\pi ki/N}$, where $k = 0, 1, \dots, N-1$. For the DFT, Parseval's theorem takes the form:

$$\sum_{i=0}^{N-1} |y_i|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |Y_k|^2, \quad (12)$$

where the factor $1/N$ appears due to the different definitions of the DFT in the forward and inverse transforms [14].

The PSD is defined as the square of the modulus of the Fourier transform: for the continuous case, $\text{PSD}(f) = |Y(f)|^2$, and for the discrete case, $\text{PSD}_k = \frac{1}{N\Delta x} |Y_k|^2$, where Δx is the sampling interval of the profile [6]. Considering equations (5) and (6), and applying Parseval's theorem to the definition of R_q , we obtain for the continuous case:

$$R_q^2 = \frac{1}{L} \int_0^L y^2(x) dx = \frac{1}{L} \int_{-\infty}^{\infty} |Y(f)|^2 df = \int_{-\infty}^{\infty} \text{PSD}(f) df. \quad (13)$$

For the discrete case, analogously:

$$R_q^2 = \frac{1}{N} \sum_{i=0}^{N-1} y_i^2 = \frac{1}{N} \sum_{k=0}^{N-1} |Y_k|^2 = \Delta x \sum_{k=0}^{N-1} \text{PSD}_k. \quad (14)$$

This result has a profound physical meaning: the square of the RMS R_q^2 , which is the variance of the profile, can be decomposed into contributions from different spatial frequencies (scales of irregularities) [7]. In other words, the integral (or sum) of the PSD over all frequencies yields the total variance of the profile, which allows for a quantitative assessment of the contribution to the total roughness made by high-frequency components (fine roughness) and low-frequency components (waviness, form) [10]. This confirms the energy interpretation of R_q as a parameter proportional to the energy of the profile, which was introduced earlier [8].

The practical significance of Parseval's theorem in surface metrology cannot be overestimated for several reasons. First, it provides a powerful tool for verifying the correctness of PSD computations: if the sum (or integral) of the computed PSD does not equal R_q^2 (taking into account scaling factors), this indicates errors in the FFT algorithm, incorrect normalisation of the PSD, or the presence of artefacts (such as outliers) in the profile [6]. This enables the automation of quality control for spectral measurements [14]. Second, the theorem allows for a quantitative assessment of the contribution of different scales of irregularities to the total roughness. For example, the contribution of roughness (frequencies above the cutoff frequency f_c) is defined as:

$$R_q^2(\text{roughness}) = \int_{f_c}^{\infty} \text{PSD}(f) df, \quad (15)$$

and the contribution of waviness (frequencies below f_c) as:

$$R_q^2(\text{waviness}) = \int_0^{f_c} \text{PSD}(f) df, \quad (16)$$

with the verification that $R_q^2 = R_q^2(\text{roughness}) + R_q^2(\text{waviness})$ [7]. Thus, Parseval's theorem justifies the possibility of separating roughness and waviness through frequency filtering, which is the basis of standard filtering procedures, in particular according to ISO 11562 (Gaussian filter) [15]. Third, Parseval's theorem is used in the calibration of roughness measuring instruments, since it establishes an absolute relationship between the profile in the spatial domain and its spectrum, which allows for the verification of the vertical scale calibration of the instrument, checking the correctness of filtering, and ensuring the metrological traceability of PSD measurements to roughness standards [9].

It is important to note that Parseval's theorem is valid for the full spectrum (from zero to infinite frequency). However, in real measurements, only a limited frequency band is available, determined, on the one hand, by the trace length (low-frequency limitation) and, on the other hand, by the stylus tip radius (high-frequency limitation) [12]. This means that the integral of the measured PSD yields not the full value of R_q^2 , but only that part of it which lies within the bandwidth of the instrument.

The application of Parseval's theorem in this case gives:

$$R_q^2(\text{bandlimited}) = \int_{f_{\min}}^{f_{\max}} \text{PSD}(f) df \leq R_q^2, \quad (17)$$

which is consistent with the discussion of bandwidth limitations in the previous section [12]. Furthermore, the use of filters (for example, the Gaussian filter) changes the spectrum of the signal; therefore, Parseval's theorem applies to the filtered signal rather than to the original one, and when comparing R_q for different filters, it must be taken into account that these are different quantities [15].

As it was said above, Parseval's theorem is closely related to other fundamental theorems of spectral analysis. In particular, the Wiener-Khinchin theorem states that the PSD is the Fourier transform $\mathcal{F}$ of the autocorrelation function (ACF): $\text{PSD}(f) = \mathcal{F}\{ACF(\tau)\}$ [7]. Together with Parseval's theorem, which relates the PSD to the variance ($R_q^2 = \int \text{PSD}(f) df$), these two theorems form the foundation of the spectral theory of random processes, which is the basis for the statistical description of surfaces [10]. In addition, the Nyquist–Shannon sampling theorem determines the conditions for signal discretization, and Parseval's theorem is necessary for the correct computation of the PSD via the DFT, since it ensures the conservation of energy during discretisation [14].

In the context of all the material presented, Parseval's theorem serves as the connecting link between the two approaches to describing surfaces – the amplitude approach (height parameters, in particular R_q) and the frequency approach (power spectral density). It ensures the mathematical unity of these descriptions, confirms the energy interpretation of roughness parameters, allows for the quantitative assessment of the contribution of different scales of irregularities, and serves as the basis for the standardisation of filtering and calibration [12]. Thus, Parseval's theorem completes the logical chain: profile $\rightarrow$ spectrum $\rightarrow$ parameters [16], ensuring the unity of the considered methods for describing surfaces and their metrological assurance [17].

The purpose of this article is to establish and demonstrate the application of Parseval's theorem to digital spectral analysis by deriving the relationship between the mean-square value (the square of the RMS amplitude) of a centered signal and the area under its one-sided power spectral density, and to investigate how sampling conditions, additive noise, and spectral estimation by Welch's method affect the numerical preservation and physical interpretation of signal power.

3. Research methodology

Let's get introduced the following mathematical symbols and units of measurement: $x(t)$ is instantaneous value of the vibration signal at time (t), [mV]; R_q is RMS amplitude of the vibration signal, [mV]; R_q^2 is square of the RMS amplitude (variance or mean power of the signal), [mV²]; T is total observation period or signal duration, [s]; $X(f)$ is amplitude spectral density (Fourier

transform of the signal), $[\text{mV} \cdot \text{s}]$ or $[\text{mV}/\text{Hz}]$; $S(f)$ is two-sided PSD, $[\text{mV}^2/\text{Hz}]$; $G(f)$ is one-sided PSD (defined only for physically real, positive frequencies), $[\text{mV}^2/\text{Hz}]$; f is cyclic frequency, $[\text{Hz}]$ ($\omega = 2\pi f$), $[\text{rad/s}]$.

3.1. Step-by-Step Mathematical Derivation

By definition from mathematical analysis and signal theory, the square of the RMS value of a zero-mean (centered, $DC = 0$) random process equals its total variance:

$$R_q^2 = \frac{1}{T} \int_0^T x^2(t) dt. \quad (18)$$

Parseval's theorem states that the total energy (or power) of a signal is identical in both the time and frequency domains. For a continuous signal over an extended period, the mean power can be expressed via its two-sided spectral density function $S(f)$, mathematically defined across the entire real frequency axis from $(-\infty)$ to $(+\infty)$ (21) [11]:

$$\frac{1}{T} \int_0^T x^2(t) dt = \int_{-\infty}^{+\infty} S(f) df. \quad (19)$$

Having established the equivalence of the mean power in the time domain and the integral of the two-sided spectral density, we now transition to a physically meaningful representation. Negative frequencies do not exist in the physical world. Because the power spectrum of a real-valued physical signal is perfectly symmetrical about zero $S(f) = S(-f)$, the energy from the negative frequency domain can be folded into the positive domain. We introduce the one-sided power spectral density $G(f)$ defined strictly for $f \geq 0$:

1. For $(f > 0)$: $G(f) = 2S(f)$ (the spectral density amplitude doubles).
2. For $f = 0$ (DC component): $G(0) = S(0)$. Given our zero-mean assumption $DC = 0$ and $G(0) = 0$.

Replacing the two-sided integral with its one-sided physical equivalent yields (22):

$$\int_{-\infty}^{+\infty} S(f) df = \int_0^{+\infty} G(f) df. \quad (20)$$

Therefore, we obtain the final equality (23):

$$R_q^2 = \int_0^{+\infty} G(f) df = AUC, \quad (21)$$

where AUC is the area under the $G(f)$ curve.

This completes the theoretical proof. The square of the RMS amplitude perfectly matches the area under the one-sided power spectral density curve.

3.2. Practical Engineering Control Case

To make these concepts intuitive, continuous integrals are replaced by discrete summations over a multi-harmonic signal. So, baseline signal configuration is the following. Let us consider a synthetic vibration signal composed of three independent harmonic components (harmonics) without any constant offset ($DC = 0$): harmonic 1: $f_1 = 50$ Hz, Amplitude $A_1 = 10$ mV; harmonic 2: $f_2 = 100$ Hz, Amplitude $A_2 = 5$ mV; harmonic 3: $f_3 = 150$ Hz, Amplitude $A_3 = 12$ mV.

4. Results and discussion

4.1. Scenario A: Sampling at Exactly the Critical Nyquist Boundary ($f_s = 300$ Hz)

The research results (Figure 1) were visualized using the Python programming language with the Matplotlib and Seaborn libraries. We have had the next solution (Fig. 1): 1) calculated R_q^2 in the time domain is equal to 62.50 mV^2 ; 2) area AUC PSD equals 62.50 mV^2 . Therefore, the difference between them is equal to zero.

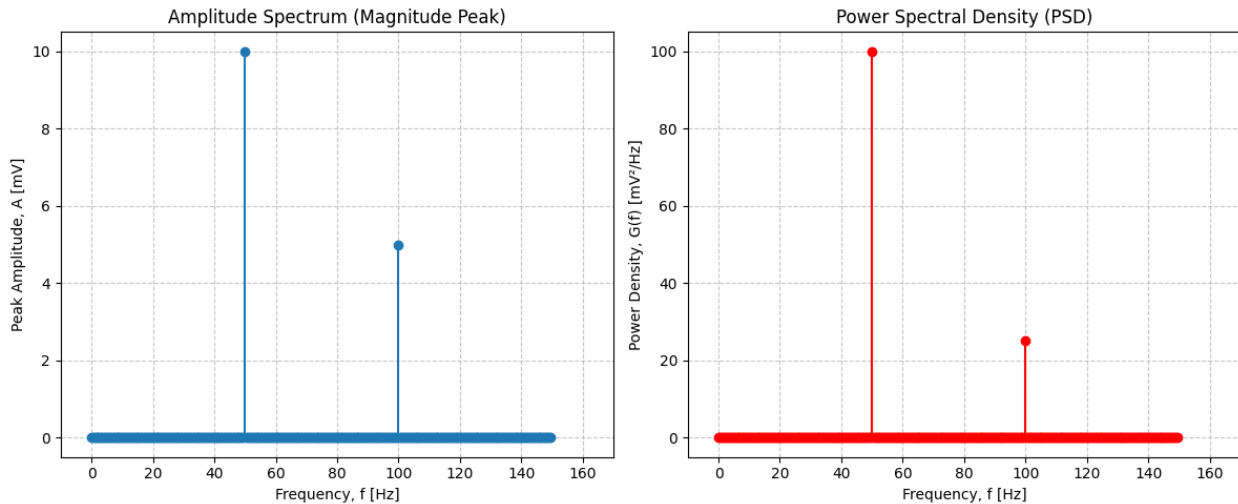


Fig. 1. Magnitude peak (left) and PSD (right)

Source: compiled by the authors

Fig. 1 shows the amplitude spectrum (left) and PSD (right) for a multi-harmonic signal sampled exactly at the critical Nyquist boundary ($f_s = 300$ Hz).

Left panel (Amplitude Spectrum): The discrete magnitude spectrum reveals only two dominant peaks at 50 Hz and 100 Hz, with amplitudes of 10 mV and 5 mV, respectively. The third harmonic component at 150 Hz, which physically has an amplitude of 12 mV, is completely absent from the spectrum. This is a direct consequence of sampling exactly at the Nyquist frequency ($f_s = 2 \cdot f_{\text{max}} = 300$ Hz), where the sampling instants coincide with the zero-crossings of the 150 Hz sine wave.

Right panel (Power Spectral Density, PSD): The PSD plot, calculated as the squared amplitude spectrum normalized by the frequency resolution (Δf), exhibits the same spectral signature. Only the 50 Hz and 100 Hz components contribute to the total area under the curve (*AUC*). Despite the visible loss of the 150 Hz component, Parseval's theorem remains strictly satisfied for the discrete sampled data: the time-domain variance ($R_q^2 = 62.50$ mV²) exactly equals the integrated area under the PSD curve ($AUC = 62.50$ mV²). However, this captures only 46.5% of the true physical signal power (134.50 mV²), demonstrating that while Parseval's equality holds for any discrete array, it does not guarantee fidelity to the underlying continuous physical process when sampling violates oversampling margins.

The Mathematical Phenomenon of Phase Dropout. When testing this scenario computationally using a pure sine wave base $\sin(2\pi \cdot 150 \cdot t)$, the third harmonic component completely vanishes from the resulting discrete frequency spectrum. At exactly $f_s = 300$ Hz, samples are recorded at uniform intervals of exactly $\Delta t = 1/300$ seconds.

Substituting these discrete timesteps into the argument of the function yields:

$$\sin\left(2\pi \cdot 150 \cdot \frac{n}{300}\right) = \sin(\pi \cdot n) = 0 \quad (\text{for all integer values of } n).$$

Because the digital measurement system captures data precisely when the sine wave crosses the zero-amplitude baseline, the physical component is rendered invisible to standard Fast Fourier Transforms (FFT).

4.2. Scenario B: Oversampling Without and With the Noise ($f_s = 600$ Hz)

4.2.1. Without the Noise ($f_s = 600$ Hz)

To avoid phase dropout at the Nyquist boundary, the sampling rate is increased to ($f_s = 600$ Hz).

Calculation in the Time Domain. For orthogonal sinusoidal waves, the total variance is equal to the sum of the mean powers of each individual component:

$$(R_q^2 = \frac{A_1^2}{2} + \frac{A_2^2}{2} + \frac{A_3^2}{2} = \frac{10^2}{2} + \frac{5^2}{2} + \frac{12^2}{2} = 50 + 12.5 + 72 = 134.5 \text{ mV}^2).$$

Signal Parameterization Matrix. When mapping out the physical attributes across a real frequency axis, the positive amplitude properties double relative to their mathematical two-sided counterparts to preserve total power (Table 1).

Table 2. Signal Parameterization Matrix

Frequency, f [Hz]	Component Type	Peak Amplitude, A [mV]	Two-Sided Power, S [mV ²]	One-Sided Power, ($P = A^2/2$)[mV ²]
0	DC Baseline	0	0	0
50	Harmonic 1	10	25.0	50.0
100	Harmonic 2	5	6.25	12.5
150	Harmonic 3	12	36.0	72.0

Source: compiled by the authors

Solution results are the following (Fig. 2). Time domain RMS R_q^2 : 134.50 mV². Frequency domain area under curve, i.e., AUC PSD equals 134.50 mV². Mathematical convergence check: true. Parseval's theorem is exactly satisfied: 134.50 mV² = 134.50 mV².

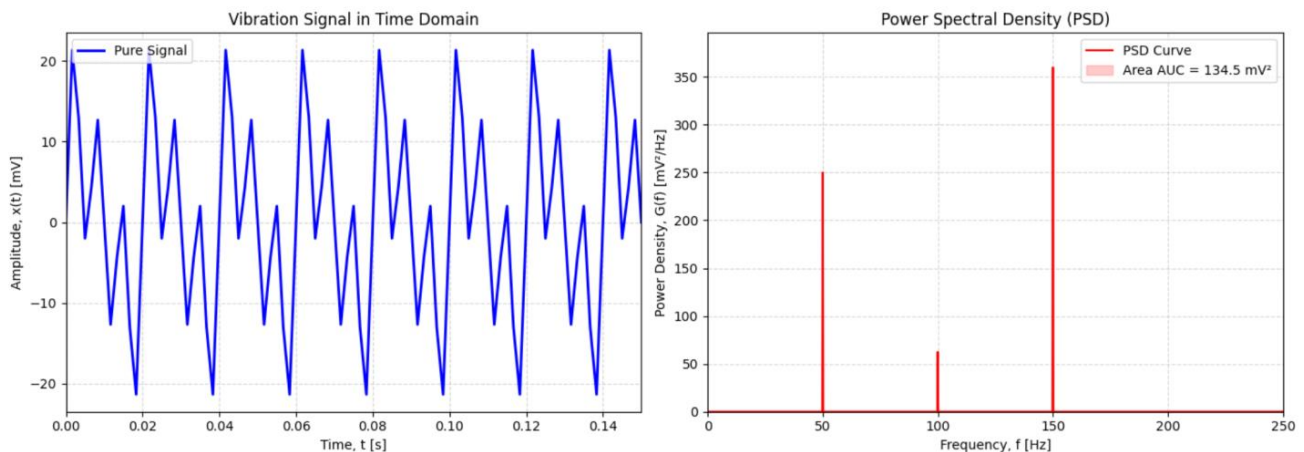


Fig. 2. Without noise: vibration signal in time domain (left) and PSD (right)

Source: compiled by the authors

4.2.2. With the Noise ($f_s = 600$ Hz, i.e., the impact of the white noise insertion)

Adding Gaussian white noise shifts the spectrum from discrete delta-spikes to a continuous profile. The noise introduces a uniform baseline across all frequency components. If a noise floor with a standard deviation of ($\sigma = 4$ mV) is injected, it adds a variance component of $\sigma^2 = 16$ mV² directly to the overall signal energy budget:

$$(R_{q,\text{total}}^2 \approx 134.5 \text{ mV}^2 + 16.0 \text{ mV}^2 = 150.5 \text{ mV}^2).$$

Solution gives the following (Fig. 3). Time domain RMS R_q^2 : 151.80 mV². Frequency domain area under curve (AUC) PSD: 151.82 mV². Mathematical convergence check: false.

4.3. Scenario C: Spectral Smoothing via Welch's Method

While the discrete FFT captures total energy accurately, computing continuous noise spectra directly over long durations compresses harmonic signals into infinitely narrow, high-amplitude spikes. This makes visual interpretation difficult.

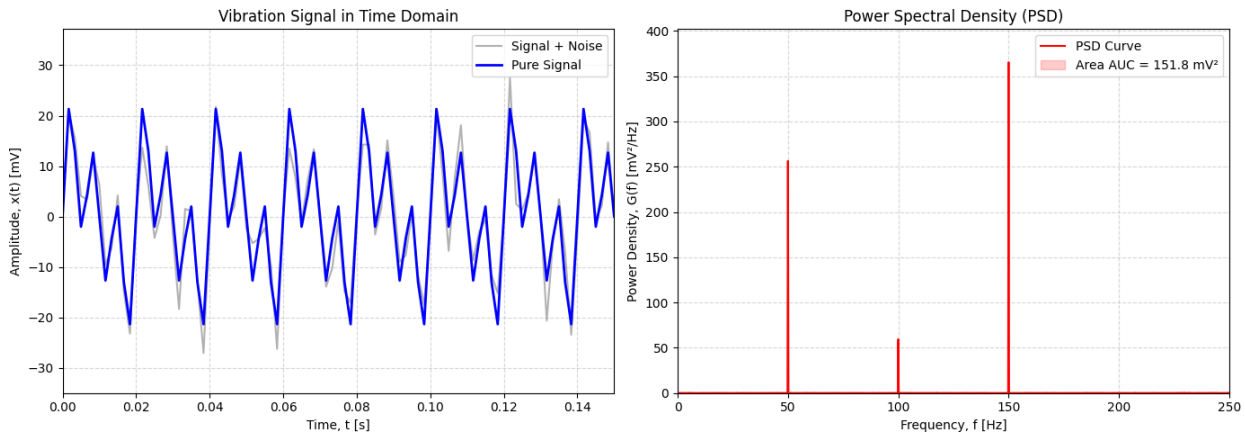


Fig. 3. With the noise: vibration signal in time domain (left) and PSD (right)

Source: compiled by the authors

Welch's method addresses this by partitioning data strings into overlapping segments, computing individual windowed periodograms, and averaging the resulting spectra. This transforms sharp spikes into smooth spectral hills. The integration of this continuous curve over frequency steps (Δf) yields a clean, visible area that corresponds perfectly to the system variance (R_q^2), providing an effective visualization for lecture presentations [10]. Solution results are the following (Fig. 4). Time domain variance Rq^2 : 159.65 mV^2 . Frequency domain area under PSD curve (i.e., AUC): 160.24 mV^2 . Mathematical convergence check: true.

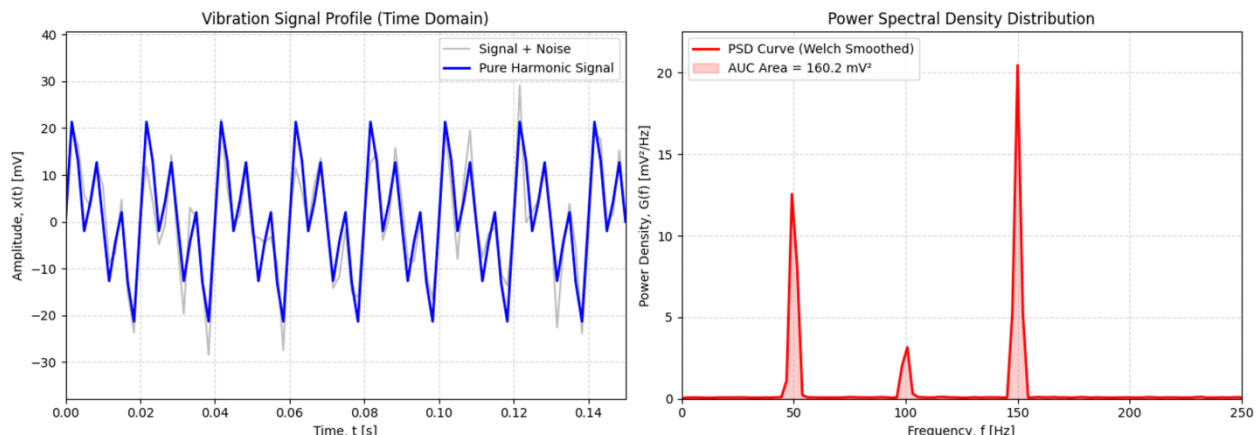


Fig. 4. Welch's method: vibrational signal in time (left) and frequency (right) domains

Source: compiled by the authors

Left Subplot Panel (Graph A): Time domain tracking plotting raw amplitude values [mV] versus operational windows [s]. Must plot both the underlying clean composite sine waves (blue) alongside the noisy tracking data (gray).

Right Subplot Panel (Graph B): Power Spectral Density distribution plotting [mV^2/Hz] across target frequency bins [Hz]. The three dominant peaks must rise at exactly 50, 100, and 150 Hz. The total integration area beneath this curve must be shaded with a semi-transparent color fill to represent the physical area under the curve (AUC).

Discussion (of the mathematical phenomenon of phase dropout)

Here is an analysis of the critical Nyquist boundary and the 46.5% power phenomenon (case 1). The experimental data obtained under the critical Nyquist boundary conditions ($f_s = 300$ Hz, Fig. 1) reveals a subtle yet profound discrepancy between continuous physical reality and discrete computational metrics. According to the time-domain synthesis, the true total power (σ_{true}^2) embedded within the original analog continuous multi-harmonic signal is defined by the sum of the standalone powers of its orthogonal constituents:

$$\sigma_{\text{true}}^2 = \frac{A_1^2}{2} + \frac{A_2^2}{2} + \frac{A_3^2}{2} = \frac{10^2}{2} + \frac{5^2}{2} + \frac{12^2}{2} = 50.0 + 12.5 + 72.0 = 134.50 \text{ mV}^2.$$

However, when evaluating the discrete frequency spectrum at a sampling rate of exactly (300 Hz), the measured cumulative power yields only 62.50 mV². This captured energy represents exactly 46.5% of the true physical signal power:

$$\text{Captured Power \%} = \frac{62.50 \text{ mV}^2}{134.50 \text{ mV}^2} \times 100 \% \approx 46.47 \% \rightarrow 46.5 \%.$$

This 53.5 % energy deficit is directly caused by the phenomenon of phase dropout (or phase cancellation) occurring precisely at the Nyquist frequency boundary. Because the highest harmonic component ($f_3 = 150 \text{ Hz}$) is sampled at intervals of exactly $\Delta t = 1/300 \text{ s}$, the discrete time-domain arguments evaluate to:

$$\sin\left(2\pi \cdot 150 \cdot \frac{n}{300}\right) = \sin(\pi \cdot n) = 0 \quad (\forall n \in \mathbb{Z}).$$

Consequently, the digital data acquisition system captures data strictly at the zero-crossing instances of the third harmonic, effectively rendering its amplitude non-existent (0 mV) within the discrete digital array.

Crucially, students and researchers must distinguish between physical fidelity and mathematical consistency. As indicated in the computational console logs, Parseval's theorem is still strictly satisfied with an absolute difference of zero. This occurs because Parseval's equality is a rigorous algebraic invariant between a discrete sequence in time and its corresponding discrete transform in frequency. Since the points sampled in the time domain already omitted the third harmonic, the time-domain variance calculation ($\text{mean}(x^2)$) yields exactly (62.50 mV²), which perfectly mirrors the discrete area under the curve (*AUC*) of the spectrum.

Therefore, while Parseval's energy conservation remains computationally flawless for any digital array, violating oversampling margins distorts the physical reality of the continuous underlying process, losing more than half of the system's actual kinetic or electrical energy budget.

5. Application to unmanned automotive systems

The developed digital signal processing (DSP) approach can be directly applied to vibration monitoring of unmanned automotive systems. In such systems, vibration measurements obtained from accelerometers installed on the chassis, suspension elements, wheel hubs, powertrain, or other structural components provide discrete time-domain signals that can be transformed into the frequency domain for condition monitoring and technical information support.

The essential processing chain can be represented as:

vibration measurement → preprocessing → sampling → DFT/FFT → PSD estimation → spectral analysis → diagnostic information.

Let $x_i = x(t_i)$ denote a discrete vibration signal obtained from an onboard sensor, where $i = 0, 1, \dots, N-1$, and let f_s be the sampling frequency. Before spectral analysis, the mean value is removed:

$$x_i^c = x_i - \bar{x}.$$

where

$$\bar{x} = \frac{1}{N} \sum_{i=0}^{N-1} x_i.$$

The time-domain mean-square value of the centered vibration signal is then determined as

$$R_x^2 = \frac{1}{N} \sum_{i=0}^{N-1} (x_i^c)^2.$$

The discrete Fourier transform provides the corresponding frequency-domain representation:

$$X_k = \sum_{i=0}^{N-1} x_i^c e^{-j2\pi ki/N}.$$

with the corresponding frequency values

$$f_k = \frac{k f_s}{N}.$$

A power spectral density (PSD) estimate $G(f_k)$ can then be obtained from the squared magnitude of the spectral components with the appropriate normalization. For a real-valued vibration signal, the one-sided PSD is considered over the frequency interval from zero to **the Nyquist frequency**, i.e.:

$$f_{\text{Nyq}} = \frac{f_s}{2}.$$

The total spectral power contained in the analyzed bandwidth is therefore obtained from the area under the PSD curve (*AUC*):

$$P_{\text{PSD}} = \int_0^{f_s/2} G(f) df.$$

The correctness of the transition from the time domain to the frequency domain is verified by Parseval's theorem. For the same discrete dataset and consistent PSD normalization, the time-domain mean-square value and the integrated one-sided PSD must satisfy:

$$R_x^2 \approx P_{\text{PSD}}.$$

For discrete numerical calculations, the verification can be expressed through the relative discrepancy

$$\varepsilon_p = \frac{|R_x^2 - P_{\text{PSD}}|}{R_x^2} \times 100 \ %.$$

Thus, the value of ε_p provides a direct internal consistency check for the digital signal-processing chain. A small discrepancy indicates numerical consistency between the time-domain and frequency-domain representations, whereas a significant discrepancy requires examination of PSD normalization, frequency resolution, integration limits, windowing, filtering, or other processing parameters.

The four scenarios investigated in this study have direct implications for unmanned automotive vibration monitoring. First, the sampling frequency must provide an adequate margin above the highest frequency component of interest. As demonstrated by Scenario A, sampling exactly at the Nyquist boundary may cause a physically existing vibration component to disappear from the sampled signal when the sampling instants coincide with its zero crossings. In this case, Parseval's equality can still be satisfied for the discrete data, while the digital representation does not reproduce the complete physical vibration signal. Consequently, mathematical consistency of the DSP procedure must not be confused with physical fidelity of the measurement.

Second, increasing the sampling frequency, as demonstrated in Scenario B, improves the representation of high-frequency vibration components and allows their contribution to the total

signal power to be retained. This is particularly important for unmanned automotive systems in which structural, suspension, wheel, and powertrain vibrations may contain components distributed over different frequency ranges.

Third, real vehicle measurements inevitably contain stochastic components and measurement noise. Scenario C demonstrates that additive Gaussian noise contributes directly to the measured mean-square power and therefore appears as an additional broadband component of the PSD. The total vibration level should consequently be interpreted together with the sensor bandwidth and noise characteristics.

Finally, Scenario D demonstrates the practical usefulness of Welch's method for vibration monitoring. Averaging modified periodograms produces a smoother PSD estimate, facilitating identification of broadband vibration regions and comparison of spectral power between operating conditions. The integrated PSD can nevertheless be checked against the time-domain mean-square value to verify the consistency of the spectral estimation procedure.

Accordingly, the proposed DSP framework provides an information-support layer for unmanned automotive systems (UASs): the time-domain vibration signal supplies the measured physical response, the frequency-domain representation identifies its spectral components, and Parseval's theorem provides a quantitative consistency check between the two domains. The approach can therefore be used as a preprocessing and verification stage for subsequent vibration-based monitoring, diagnostics, and condition-assessment algorithms.

6. Conclusions

1. The study demonstrates that Parseval's theorem provides a rigorous quantitative relationship between the mean-square value of a centered signal in the time domain and the area under its one-sided power spectral density in the frequency domain. This relationship can therefore be used as an internal consistency criterion for digital spectral calculations and for verifying the transition from time-domain vibration measurements to frequency-domain representations.

2. The sampling investigation demonstrates that mathematical consistency does not necessarily imply physical fidelity. At the sampling frequency of 300 Hz, the 150 Hz harmonic coincides with the Nyquist boundary and disappears from the sampled data because the sampling instants coincide with its zero crossings. The time-domain mean-square value and the integrated power spectral density both equal 62.50 mV^2 , satisfying Parseval's equality exactly, whereas the original continuous signal contains 134.50 mV^2 . Thus, only 46.5% of the original signal power is retained in the sampled data. Increasing the sampling frequency to 600 Hz restores the 150 Hz component and yields complete agreement between the time-domain and frequency-domain values of 134.50 mV^2 .

3. The introduction of Gaussian noise increases the measured mean-square power and produces a broadband spectral component. In the investigated case, the time-domain value was 151.80 mV^2 , whereas integration of the power spectral density gave 151.82 mV^2 , with a discrepancy of approximately 0.01%. Welch's method produced a smoother spectral representation; its corresponding values were 159.65 mV^2 in the time domain and 160.24 mV^2 from the integrated spectrum, with a difference of less than 0.4%. These results confirm that spectral smoothing changes the visual structure of the spectrum while maintaining close agreement of the total mean-square power.

4. The results clarify the complementary roles of Parseval's and Wiener–Khinchin theorems. Parseval's theorem provides the quantitative equality between mean-square quantities in the time and frequency domains, whereas the Wiener–Khinchin theorem relates the power spectral density of a stationary random process to the Fourier transform of its autocorrelation function. This distinction is important when moving from deterministic signal analysis to stochastic vibration measurements containing noise and other random components.

5. The developed approach provides a practical information-support framework for vibration monitoring of unmanned ground vehicles. The measured vibration signal can be subjected to preprocessing, sampling, Fourier transformation, power spectral density estimation, and spectral

analysis, while Parseval's theorem provides an independent consistency check of the resulting frequency-domain representation. The study therefore establishes a methodological basis for subsequent vibration-based monitoring, diagnostics, and condition assessment of unmanned ground vehicles, while demonstrating that adequate sampling conditions and measurement bandwidth are prerequisites for preserving physically meaningful information.

6. The same principle can subsequently be extended to other signals associated with unmanned ground vehicle operation, including vehicle-body accelerations, suspension responses, drivetrain oscillations, road-surface irregularities, and control-system errors. Therefore, the spectral approach considered in this study provides a general information-processing framework that can be extended from signal analysis and condition monitoring toward the development of intelligent monitoring and adaptive control systems for unmanned ground vehicles including artificial intelligence.

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Спектральне інформаційне забезпечення безпілотних наземних транспортних засобів

Ларшин Василь Петрович¹⁾

ORCID: <https://orcid.org/0000-0001-7536-3859>; vasilylarshin@gmail.com. Scopus Author ID: 7801669969

Шаповалов Денис Олегович¹⁾

ORCID: <https://orcid.org/0009-0007-4125-8875>; 388839@as.op.edu.ua

Кішнян Ігор Володимирович²⁾

ORCID: <https://orcid.org/0000-0001-7838-5607>; myhouse72@ukr.net

Злепко Ігор Юрійович²⁾

ORCID: <https://orcid.org/0009-0002-2582-0083>; zlepko@meta.ua

Мальцев Олександр Васильович²⁾

ORCID: <https://orcid.org/0000-0001-7377-5972>; mav1946fish@gmail.com

¹⁾ Національний університет «Одеська політехніка», пр. Шевченка, 1. Одеса, 65044, Україна

²⁾ Військова академія (м. Одеса), Фонтанська дорога, 10. Одеса, 65009, Україна

АНОТАЦІЯ

У статті досліджується застосування теореми Парсеваля до цифрового опрацювання сигналів та її використання для встановлення зв'язку між середньоквадратичним значенням сигналу та інтегралом його спектральної щільності потужності. Особливу увагу приділено співвідношенню між квадратом середньоквадратичної амплітуди та площею під кривою односторонньої спектральної щільності потужності, яка визначає загальну середньоквадратичну потужність сигналу в досліджуваному частотному діапазоні. Для дослідження впливу умов дискретизації, адитивного шуму та методів спектрального оцінювання на збереження й інтерпретацію потужності сигналу використано контрольований багатогармонічний сигнал. Розглянуто декілька обчислювальних випадків, що дають змогу проаналізувати вплив дискретизації на межі Найквіста, підвищення частоти дискретизації, стохастичного шуму та спектрального згладжування методом Велча. Результати демонструють, що теорема Парсеваля забезпечує точне співвідношення узгодженості

потужності для дискретного сигналу, представленого вибірковими даними, однак сама по собі ця узгодженість не гарантує адекватного відтворення початкового неперервного фізичного сигналу. Тому для фізично змістовного спектрального аналізу необхідними є належна частота дискретизації, смуга вимірювання та обґрунтована методологія спектрального оцінювання. Показано, що інтеграл спектральної щільності потужності може використовуватися як кількісний критерій перевірки узгодженості розрахунків у часовій та частотній областях. Уточнено співвідношення між теоремою Парсеваля та теоремою Вінера–Хінчина шляхом розмежування збереження середньоквадратичних величин і статистичного опису стаціонарних випадкових процесів. Розроблений підхід до опрацювання сигналів розглянуто також як інформаційне забезпечення вібраційного моніторингу безпілотних наземних транспортних засобів, у якому виміряні вібраційні сигнали перетворюються у частотні представлення для подальшого моніторингу, діагностування та оцінювання технічного стану. Теорема Парсеваля при цьому використовується як внутрішній критерій перевірки коректності переходу від вимірних сигналів у часовій області до їх спектрального представлення.

Ключові слова: цифрова обробка сигналів; теорема Парсеваля; спектральна щільність потужності; середньоквадратичне значення; межа Найквіста, випадання фази; метод Велча; площа під кривою; безпілотний наземний транспортний засіб

ІНФОРМАЦІЯ ПРО АВТОРІВ



Vasily P. Larshin - Academician of the Ukrainian Academy of Economic Cybernetics. Dr. Sc. (Eng.), Professor, Department of Machine Dynamics and Mechanical Engineering. Odesa Polytechnic National University, 1, Shevchenko Av. Odesa, 65044, Ukraine
ORCID: <https://orcid.org/0000-0001-7536-3859>; vasilylarshin@gmail.com. Scopus Author ID: 7801669969
Research field: mechatronic systems in manufacturing and automotive engineering

Ларшин Василь Петрович - академік Української академії економічної кібернетики, докт. техн. наук, професор кафедри Динаміки машин та механічної інженерії. Національний університет «Одеська політехніка», пр. Шевченка, 1. Одеса, 65044, Україна

Галузь досліджень: мехатронні системи у технології машинобудування і автомобільній техніці



Denys O. Shapovalov - PhD student, Department of Automobile Transport and Logistics. Odesa Polytechnic National University, 1, Shevchenko Av. Odesa, 65044, Ukraine
ORCID: <https://orcid.org/0009-0007-4125-8875>; 388839@as.op.edu.ua
Research field: mechatronic CNC machine tools configured by control

Шаповалов Денис Олегович - аспірант кафедри Автомобільного транспорту та логістики. Національний університет «Одеська політехніка», пр. Шевченка, 1. Одеса, 65044, Україна

Галузь досліджень: мехатронні верстати з ЧПК, що конфігуруються управлінням



Ihor V. Kishianus - Senior Lecturer of the Department of Military Vehicle Equipment. Military Academy (Odesa-city), Fontans'ka doroha, 10, 65009, Ukraine
ORCID: <https://orcid.org/0000-0001-7838-5607>; myhouse72@ukr.net
Research field: Modern vehicle operation technologies

Кішіанус Ігор Володимирович - старший викладач кафедри Автомобільної техніки. Військова академія (м. Одеса), Фонтанська дорога, 10, 65009, Україна

Галузь досліджень: сучасні технології експлуатації автомобільної техніки



Igor Yu. Zlepko - Head of the Automotive Equipment Service Group of the Department of Military Vehicle Equipment. Military Academy (Odesa-city), Fontans'ka doroha, 10, 65009, Ukraine
ORCID: <https://orcid.org/0009-0002-2582-0083>; zlepko@meta.ua
Research field: Modern vehicle operation technologies

Злепко Ігор Юрійович - керівник групи Обслуговування автомобільної техніки кафедри автомобільної техніки. Військова академія (м. Одеса), Фонтанська дорога, 10, 65009, Україна

Галузь досліджень: сучасні технології експлуатації автомобільної техніки



Oleksandr V. Maltsev – Dr. Sc. (Eng.), Professor, Department of Motor Transport Support Organization and Automotive Training. Military Academy (Odesa-city), Fontans'ka doroha, 10, 65009, Ukraine
ORCID: <https://orcid.org/0000-0001-7377-5972>; mav1946fish@gmail.com
Research field: Modern motor transport support organization

Мальцев Олександр Васильович – д-р техн. наук, професор кафедри Організації автомобільного забезпечення та підготовки автомобілістів. Військова академія (м. Одеса), Фонтанська дорога, 10, 65009, Україна

Галузь досліджень: сучасна організація автомобільного забезпечення